

# QUANTIFYING THE RELATIONSHIP BETWEEN DRAINAGE NETWORKS AT HILLSLOPE SCALE AND PARTICLE SIZE DISTRIBUTION AT PEDON SCALE

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## Abstract

Nowadays, translating information about hydrologic and soil properties and processes across scales has emerged as a major theme in soil science and hydrology, and suitable theories for upscaling or downscaling hydrologic and soil information are being looked forward. The recognition of low-order catchments as self-organized systems suggests the existence of a great amount of links at different scales between their elements. The objective of this work was to research in areas of homogeneous bedrock material, the relationship between the hierarchical structure of the drainage networks at hillslope scale and the heterogeneity of the particle-size distribution at pedon scale. One of the most innovative elements in this work is the choice of the parameters to quantify the organization level of the studied features. The fractal dimension has been selected to measure the hierarchical structure of the drainage networks, while the Balanced Entropy Index (BEI) has been the chosen parameter to quantify the heterogeneity

of the particle-size distribution from textural data. These parameters have made it possible to establish quantifiable relationships between two features attached to different steps in the scale range. Results suggest that the bedrock lithology of the landscape constrains the architecture of the drainage networks developed on it and the particle soil distribution resulting in the fragmentation processes.

*Keywords:* Fractal Dimension; Drainage Networks; Balanced Entropy Index; Particle-Size Distribution; DEM; GIS.

## 1. INTRODUCTION

Drainage networks represent how water moves through the Earth's surface, and their structures are controlled by environmental factors such as climate, geology, and vegetation.<sup>1,2</sup> The study of the drainage networks has implications in fields as diverse as ecology,<sup>3,4</sup> civil engineering,<sup>5</sup> or planetary sciences,<sup>6</sup> and its characterization, inventory, and understanding will be essential for an integrated management of the water resources in future, and also, to evaluate potential changes to environmental resources caused by changes in the land use or climate.

The development of specific algorithms for the automatic extraction of drainage networks from Digital Elevation Models (DEMs)<sup>7,8</sup> and the increasing resolution and availability of DEMs<sup>9,10</sup> have improved and facilitated its study.

According to the erosion cycle, a drainage system can be considered an open system which is progressively losing potential energy and mass.<sup>11</sup> This loss of energy and material does not occur in an anarchic way, but occurs in a self-organized manner,<sup>12</sup> generating hierarchical structures as the natural drainage networks. The recognition of this hierarchical behavior and the hierarchical ordering of the streams which compose the drainage system in the second third of the 20th century allowed significant advances in hydrology and geomorphology fields.<sup>13–15</sup> A similar step forward in the hydrologic knowledge as represented by these works is now requested by hydrologists who, unable to generalize the findings of the current hydrological models to ungauged areas, are in the ongoing quest for physically realistic catchment-scale models, that include more appropriate representations of the heterogeneous hydrological processes.<sup>16,17</sup>

To consider low-order catchments as self-organized systems suggests that their elements must be linked across scales. Nowadays, translating

information about hydrologic and soil properties and processes across scales has emerged as a major theme in soil science and hydrology, and suitable theories for upscaling or downscaling hydrologic and soil information are being looked forward.<sup>18</sup> Finding quantitative relationships between factors as soil texture and structure, hydromorphology, preferential flow and water movement is a crucial goal for hydrogeology.<sup>19</sup>

Nevertheless, several works have already explored the relationship between hydrological features as the drainage networks and the environmental factors of the landscape at different scales. Drainage density is a traditional measure characterizing drainage networks that has been employed in many of these works. Drainage density has been related to slope steepness,<sup>20</sup> local relief,<sup>21</sup> bedrock geology,<sup>22</sup> bedrock erodibility<sup>23</sup> and hydraulic conductivity of the underlying soil.<sup>24,25</sup> Literature shows that this last variable has been largely related to soil texture and various models have been developed to estimate hydraulic conductivity from textural data.<sup>26–29</sup>

Recently, the relationship between lithology and the hierarchical architecture of drainage networks has been shown, and quantified by means of fractal dimension.<sup>30</sup> On the other hand, fragmentation<sup>31</sup> and the erosion processes of different types of rocks are expected to generate different soil textures. It suggests the possibility of finding quantitative relationship between both last factors. Even the finding of weak relationship for factors so remote in the scale range at which they are attached might deserve the study.

The view of a drainage basin as a self-organized structure may help in choosing what quantifiable features might be related. This is a key point under the methodological point of view that this work aims to emphasize. The fractal dimension of a drainage network is seen as a measure of the complexity or level of organization at the hillslope scale. In the same manner, the point now is to research

how to measure the level of organization of soil texture at the pedon scale.

After that, the huge number of intermittent interactions along the scales makes it wise to expect that certain organization level has been created, and it might be related with the fractal dimension of the network. On the other hand the theory of complex systems, and particularly the Prigogine theory on dissipative structures,<sup>32</sup> affirms that the organization level, which is tied with the concept of “heterogeneity” or “entropic level”, may be measured by means of the Shannon’s entropy.

Shannon’s entropy<sup>33,34</sup> is an information-theoretical parameter that may be suitably interpreted as a measure of the complexity of a distribution. In fact, entropy has already been proposed in life sciences as a plausible measure of biodiversity<sup>35</sup> in the sense of evenness or heterogeneity of the diversity of species in an ecosystem. The Balanced Entropy Index (BEI) was introduced by Martín *et al.*,<sup>36</sup> and it provides an efficient way of evaluating particle-size distribution (PSD) heterogeneity from textural data.

The objective of this work was to research, in areas of homogeneous bedrock material, the relationship between the hierarchical structure of the drainage networks at hillslope scale and the surficial soil texture at pedon scale. The quantification of particle size heterogeneity is made by means of the BEI cited above.

## 2. MATERIALS AND METHODS

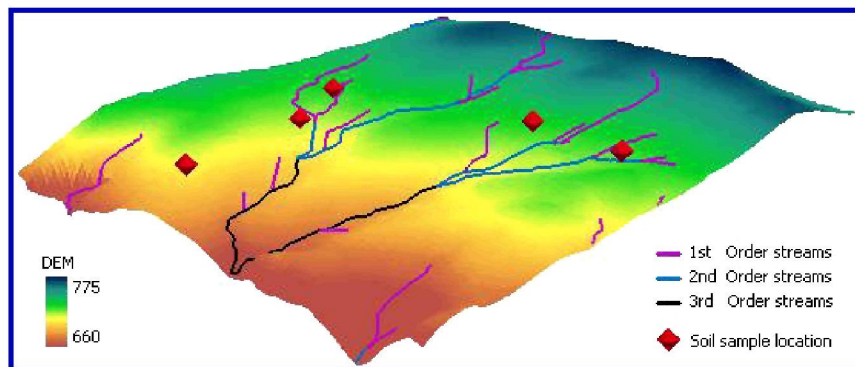
### 2.1. Dataset

This study was carried out using two sources of information: the soil map of the region, scale 1:25,000,<sup>37</sup> and the medium resolution DEM. Soil

texture included in the soil map cited above was determined by Bouyoucos method.<sup>38</sup> DEM data come from the Instituto Geográfico Nacional de España® (IGN), and correspond to a 5 m-resolution DEM generated from LiDAR data with a density of 0.5 point/m.<sup>2</sup> Altimetric accuracy of the DEM has a root mean square deviation equal to or less than 0.5 m. These DEM data are freely available under registration on the website of the IGN ([www.ign.es](http://www.ign.es)).

### 2.2. Study Area

Eight square areas (1280 m × 1280 m), located within the Spanish wine growing region called “Arribes”, have been selected to accomplish this study. The choice of the study sites was based on two criteria; first the study site must be entirely located in areas of homogeneous lithology according the lithological information included in the soil map 1:25,000, and secondly it should include analytical data of more than three surficial soil samples of the soil map database (Fig. 1). The eight study sites are distributed on two different lithologies dated in the Pre-Cambrian period, four of them [P1, P2, P3, and P4] are located on plutonic rocks, specifically on granite two-mica medium-grained, and the other four [M1, M2, M3, and M4] are situated on metamorphic rocks, particularly on pelitic and psammitic metasediments. The coordinates of the centroid of the eight study sites are provided in Table 1. The soils of the study sites are majorly entisols, and more specifically Xerorthent typic, dystric, or lithic. Associated with the drainage streams there have been described Xerofluvent typic and mollic. Only two inceptisols have been considered in this work, a Dystroxerept typic within the study site M1 and a Haploxerept typic within M2.



**Fig. 1** 3D representation (Z factor = 3) of the study site P3, showing the 5 m-resolution DEM, the drainage networks with their streams hierarchically ordered, and the location of the surficial soil samples.

**Table 1** Coordinates of the Centroid of the Study Sites. Projected Coordinate System WGS84 UTM Zone 29.

Coordinates	M1	M2	M3	M4	P1	P2	P3	P4
X UTM	699499	702420	692488	682807	732978	738752	741742	745138
Y UTM	4567003	4565032	4552425	4541370	4596071	4598618	4597113	4591706

The main climatic differences in the study area are determined by a very pronounced altitudinal thermal gradient.<sup>39</sup> According to the Köppen–Geiger climate classification<sup>40</sup> the low lands of the region, that represent a system of deep valleys, are classified as Csa (Temperate climate with dry and hot summer) while the high lands in the peneplain are classified as Bsk (Arid climate, steppe cold). The distribution of the mesoclimatic zones within the region<sup>39</sup> indicates that the study site P2 is located between both climate classes, P4 is situated majorly in the climate class Bsk and the other study sites are entirely located in the climate class Csa.

### 2.3. Method for Drainage Network Extraction

The Hydrology Toolbox included in the software ArcGIS 10.1, which was developed by the University of Texas,<sup>41</sup> was used to obtain the drainage network from the 5-m LiDAR-DEM. The drainage network extracting method in ArcGIS is based on the maximum gradient method, also called D8 algorithm, which was proposed by O`Callaghan and Mark.<sup>7</sup>

A constant area of 20,000 m<sup>2</sup> has been established as flow accumulation threshold value to initiate the channelization during the process of drainage networks extraction. It is noteworthy that all the drainage networks obtained for the study sites with this threshold area correspond to intermittent watercourses.

### 2.4. Brief Geomorphological Description

The medium resolution DEM has also been used to obtain geomorphological information and to calculate some topographic indices traditionally related to certain characteristics of the drainage networks as the drainage density. A brief analysis of elevations and slope angles has been carried out for the study sites, as well as the calculation of the Hypsometric Integral (HI).<sup>42</sup> The HI values have

been calculated by generating the elevation histogram from the 5 m DEM and using the following expression

$$HI = \frac{(H_{\text{median}} - H_{\text{min}})}{(H_{\text{max}} - H_{\text{min}})},$$

where  $H_{\text{median}}$  is the median of the distribution of DEM pixels values of the study site,  $H_{\text{min}}$  is the lowest value, and  $H_{\text{max}}$  is the highest value.<sup>43</sup>

Additionally, two properties of the drainage networks of the study sites have been investigated: the drainage density and the highest Strahler's order of the network.<sup>15</sup>

### 2.5. Fractal Analysis Methods

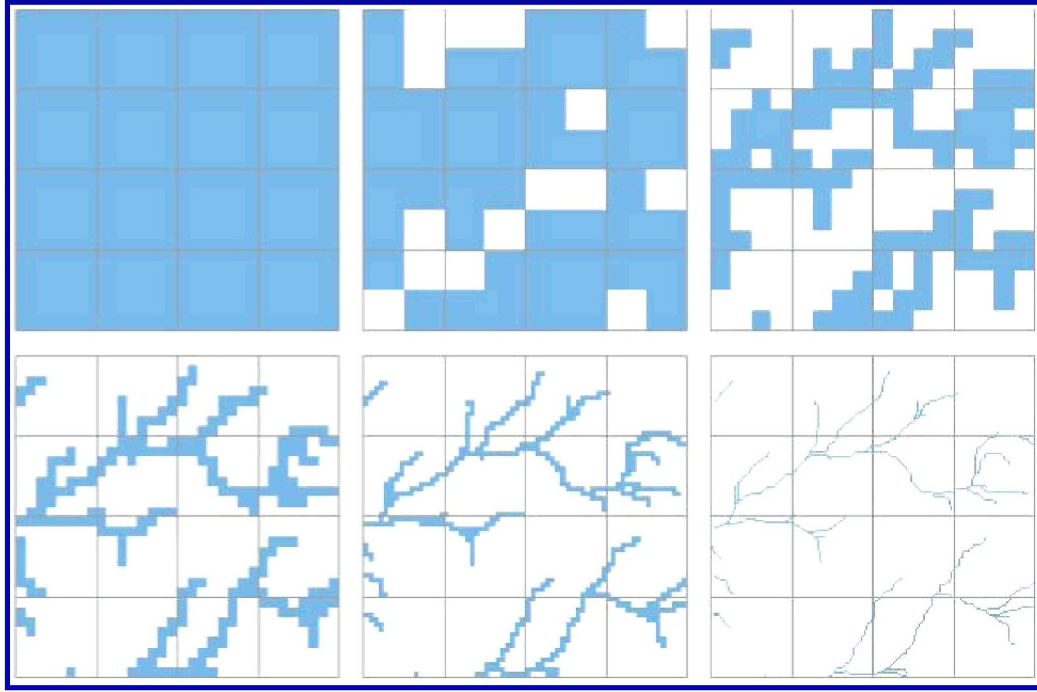
Three fractal parameters have been estimated in this work; the fractal dimension ( $D$ ) and the lacunarity ( $\Lambda$ ) of the drainage networks and the BEI of the soil particle-size distribution.

The fractal dimension of the drainage networks has been estimated using a fixed-size algorithm, the box-counting method.<sup>44</sup> The method has been applied as it was described by Rodríguez-Iturbe and Rinaldo.<sup>12</sup>

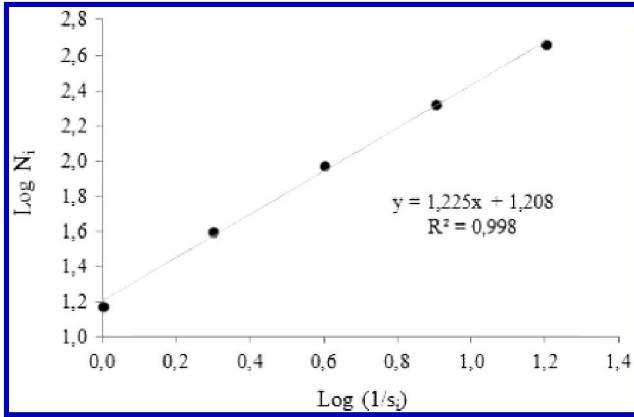
The drainage network is covered by a grid of side  $r$ . In this work, the initial grid is only one cell of side  $r = 1280$  m that matches to the study site. The number of grid boxes containing the network  $N(r)$  is counted. The value of  $r$  is halved repeatedly and, by counting the boxes crossed by the network, a series of  $N_i$  values is obtained (Fig. 2). Once a reference box size ( $r_1$ ) is established, the size factor is calculated as  $s_i = r_i/r_1$ . As  $r$  decreases to a threshold value the following relation

$$\frac{\log N_i}{\log(\frac{1}{s_i})},$$

converges to a finite value defined as box counting dimension. In this work, the reference box size was 320 m ( $r_1 = 320$  m), and the other sizes considered for the estimation were  $r_2 = 160$  m,  $r_3 = 80$  m,  $r_4 = 40$  m, and  $r_5 = 20$  m (Fig. 3).



**Fig. 2** Box-counting method applied to the drainage network of the study site M3. From left to right and up down, grid boxes containing the drainage network with box sizes of 320, 160, 80, 40, 20 m, and the original drainage network automatically generated from the 5 m-resolution DEM.



**Fig. 3** Log-log plot of the number of boxes that intersect the drainage network ( $N_i$ ) versus the size factor of the intersected boxes ( $1/s_i$ ) for the drainage network of the study unit P4.

The lacunarity of the drainage networks was obtained according to the method described by Borys.<sup>45</sup> For lacunarity estimation, the histogram of empty grid boxes of side  $r$  within grid boxes of side  $R$  needs to be calculated ( $R > r$ ). In this work,  $R = 320$  m and  $r = 20$  m, agreeing this last value with the smaller grid box size used in the fractal dimension estimation. Once the histogram is obtained, lacunarity is characterized by statistical moments, defining lacunarity as the ratio of two

expectation values,

$$\Lambda = \frac{E[X^2]}{E[X]^2},$$

where  $X$  is the variable describing the number of empty grid boxes of size 20 m within the grid boxes of size 320 m.

The BEI introduced by Martín *et al.* is given by the simple formula

$$BEI = \frac{\sum_i P_i \log P_i}{\sum_i P_i \log l_i},$$

where the numbers  $l_i$  are the lengths of the three basic size intervals ( $l_1 = 0.001$ ,  $l_2 = 0.024$  and  $l_3 = 0.975$ ) and  $P_i$ 's are the soil's clay-silt-sand fractions. The formula above can be easily computed from conventional textural data and thus provides an efficient and straightforward way of evaluating PSD heterogeneity from textural data. Additional analyses and studies are still to be performed to evaluate the BEI suitability in the estimation of variables related to PSDs as, for instance, the specific surface area which is critical for modeling geochemical processes in soils.

The BEI takes values between 0 and 1, and its value may be interpreted as follows: the bigger the value of BEI the more heterogeneous the soil's PSD and in turn the richer the soil's texture. Moreover,

since BEI may take any value from 0 to 1, entropy dimension supplies a continuum of textural classes, which is capable of a finer discrimination of soil textures, when compared with standard classifications. A thorough interpretation of the above formula and its theoretical properties in terms of texture analysis is given by Martín *et al.*<sup>36</sup>

## 2.6. Multivariate Analysis

Aiming to analyze the interactions between the studied soil fractal parameters and properties and the hydrogeological fractal parameters and properties a multivariate statistical analysis has been carried out. A total of 11 variables have been considered in the analysis; the average altitude of the study sites (Altitude average), the average slope angle of the study sites (Slope angle average), the HI of the study sites, the drainage density of the drainage network of the study sites (Drainage density), the Strahler's order of the drainage network of the study sites (Order), the fractal dimension of the drainage network of the study sites (D), the lacunarity of the drainage network of the study sites ( $\Lambda$ ), the average value of the BEI of the set of soil samples within the study site, the average clay content of the set of soil samples within the study site (Clay), the average silt content of the set of soil samples within the study site (Silt), and the average sand content of the set of soil samples within the study site (Sand).

The statistical analysis for each variable has been realized, as well as the analysis of correlations between variables, and the principal components analysis. All these operations have been carried out by using the software Statgraphics Centurion XVI.I.

All confidence intervals in this work are shown at the 95% confidence level.

## 3. RESULTS

### 3.1. Quantitative Geomorphological Approach to the Study Sites

Five variables describing the topography and the drainage system of the study sites are provided in Table 2. For both the lithologies (*M* and *P*), the widest slope angle range and the highest slope angle average appear in the study site with the widest altitude range, M4 and P3, respectively. Average slope angle is higher in metamorphic rock units ( $13.2 \pm 6.0$ ) than in plutonic rock units ( $5.4 \pm 2.7$ ). The HI values reported for the metamorphic units ( $0.65 \pm 0.09$ ) are little higher than those reported for the granitic units ( $0.57 \pm 0.10$ ) but there are not significant differences. A weak positive relationship ( $R^2 = 0.232$ ) appears between the average slope angle and the HI values. This relationship is not statistically significant but it suggests a trend. The relationship is stronger considering only the plutonic rock units ( $R^2 = 0.806$ ) while it almost disappears for the metamorphic rock units ( $R^2 = 0.001$ ) due to the values obtained for the M4 unit. Extracting this unit of the analysis, the relationship between the average slope angle and the HI values for the metamorphic units is clear ( $R^2 = 0.974$ ).

A negative linear relationship ( $R^2 = 0.427$ ) is shown between the slope average angle and the drainage density. This relationship is also not statistically significant, but it increases with removing

**Table 2** Quantitative Description of the Topography and the Drainage Network of the Study Sites.

Study Unit	Altitude Range (m.a.s.l.)	Altitude Average (m.a.s.l.)	Slope Angle Range (%)	Slope Angle Average (%)	Hypsometric Integral (HI)	Drainage Density (km/km <sup>2</sup> )	Highest Strahler's Order
M1	531–693	650	0–63	12.3	0.7346	4.28	3
M2	586–692	662	0–57	11.1	0.7170	3.81	4
M3	573–653	616	0–39	7.5	0.5375	4.96	3
M4	389–569	498	0–168	21.9	0.6056	3.26	2
P1	741–773	758	0–18	2.9	0.5313	3.78	3
P2	708–792	762	0–33	6.8	0.6429	4.65	3
P3	661–773	735	0–153	8.6	0.6607	4.18	3
P4	755–796	773	0–31	3.2	0.4390	5.07	3



Table 3 Box-Counting Results for the Drainage Network of the Study Sites.

Study Unit	Fractal Dimension ( $D$ )	Coefficient of Determination for $D$ Estimation ( $R^2$ )	Lacunarity ( $\Lambda$ )	Number of Grid Boxes Occupied by the Drainage Network with Different Boxes Sizes				
				320 m	160 m	80 m	40 m	20 m
M1	1.1897	0.9903	1.004402	14	43	96	195	406
M2	1.1368	0.9965	1.003632	15	39	85	176	363
M3	1.1927	0.9920	1.003995	16	49	105	222	469
M4	1.0913	0.9970	1.002282	15	38	76	158	323
P1	1.1459	0.9967	1.005060	14	35	81	168	339
P2	1.1824	0.9946	1.004379	16	45	102	209	447
P3	1.1602	0.9944	1.004064	15	43	89	188	400
P4	1.2250	0.9980	1.005942	15	40	93	209	458

the unit P1 of the analysis ( $R^2 = 0.821$ ). The values of drainage density for the metamorphic units ( $4.08 \pm 0.71$ ) are similar to the values obtained for the plutonic units ( $4.42 \pm 0.55$ ).

### 3.2. Box-Counting Results of the Drainage Networks Analysis

Results of the fractal dimension ( $D$ ) and the lacunarity ( $\Lambda$ ) of the drainage networks of the study sites are shown in Table 3. The suitability of the method to estimate the  $D$  of the drainage networks is supported by the high values of the coefficient of determination ( $R^2$ ) of the estimation, which are higher than 0.990 in all cases. The  $D$  of the drainage system of the metamorphic units ( $1.1526 \pm 0.0473$ ) is lower than  $D$  of the plutonic rock units ( $1.1784 \pm 0.0338$ ). This behavior was already observed in other locations of the same region in the work of Cámara *et al.*<sup>30</sup>  $\Lambda$  of the drainage system developed on metamorphic rocks ( $1.0036 \pm 0.0009$ ) is lower than  $\Lambda$  of the plutonic rock units ( $1.0049 \pm 0.0008$ ). Figure 4 shows the relationship between  $\Lambda$  and  $D$  for both lithologies. These positive relationships contradicts the assumption that lacunarity and fractal dimension are inversely related and suggest that both the parameters could be controlled by the same environmental factors.  $D$  is also closely related to the calculated drainage density. Considering all the study sites, both variables show a positive linear relationship with a coefficient of determination  $R^2 = 0.901$ . Differentiating by lithologies, the relationship is stronger in the granitic sites ( $R^2 = 0.943$ ) than in the metamorphic sites ( $R^2 = 0.867$ ).

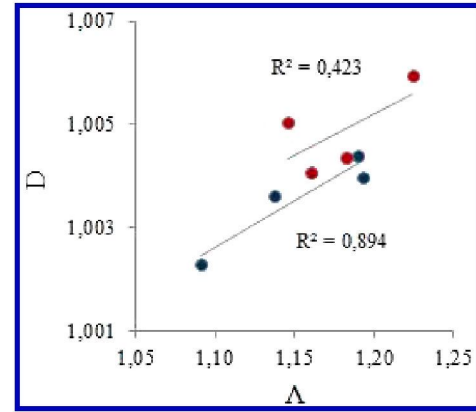


Fig. 4 Plots of the lacunarity ( $\Lambda$ ) versus the fractal dimension ( $D$ ) of the drainage networks of the study sites, showing the metamorphic lithologies (blue points) and the plutonic lithologies (red points).

### 3.3. BEI of the Particle-Size Distribution

Results have shown that the BEI averages  $0.496 \pm 0.074$  for the 27 soil samples collected on metamorphic lithologies and  $0.543 \pm 0.075$  for the 18 soil samples of the plutonic lithologies. By aggregating the results of the entropy dimension of the soil samples by study sites (Table 4), it is observed that the plutonic lithologies have a more uniform behavior ( $0.543 \pm 0.010$ ) than the metamorphic lithologies ( $0.493 \pm 0.056$ ). According to Martín *et al.*,<sup>36</sup> the majority of the obtained BEI values correspond to the sandy loam class of the United States Department of Agriculture (USDA) soil texture triangle. One of the advantages described in the work cited above using the BEI is represented by the suitability of the quantitative index to affine the textural classification. Although BEI values from 0.4

**Table 4** BEI for the Particle-Size Distribution of the Surficial Soil Samples of the Study Sites.

Study Unit	Number of Samples	BEI	Sand (%)	Silt (%)	Clay (%)
M1	9	$0.4738 \pm 0.0268$	$58.0 \pm 4.4$	$30.2 \pm 4.5$	$11.8 \pm 1.7$
M2	7	$0.5605 \pm 0.0465$	$73.0 \pm 7.5$	$17.8 \pm 7.7$	$9.2 \pm 2.5$
M3	5	$0.4254 \pm 0.0422$	$49.4 \pm 7.3$	$39.6 \pm 5.4$	$11.0 \pm 3.7$
M4	6	$0.5131 \pm 0.0673$	$68.8 \pm 6.8$	$16.3 \pm 3.0$	$14.9 \pm 6.5$
P1	4	$0.5559 \pm 0.0447$	$73.8 \pm 7.3$	$15.0 \pm 5.6$	$11.3 \pm 2.4$
P2	5	$0.5451 \pm 0.0569$	$70.6 \pm 7.9$	$17.8 \pm 4.7$	$11.6 \pm 4.2$
P3	5	$0.5341 \pm 0.0138$	$67.0 \pm 2.4$	$23.0 \pm 2.4$	$10.1 \pm 0.1$
P4	4	$0.5373 \pm 0.1511$	$63.8 \pm 22.4$	$26.3 \pm 15.7$	$10.0 \pm 6.9$

to 0.6 are all located in the sandy loam class, values between 0.5 and 0.6 are shown to have greater contents of sand than soil samples with BEI values ranging from 0.4 to 0.5. Attending to our results we can affirm that fragmentation processes on pelitic and psammitic metasediments generate finer surficial soil texture than same processes on granitic rocks.

### 3.4. Findings Across Scales

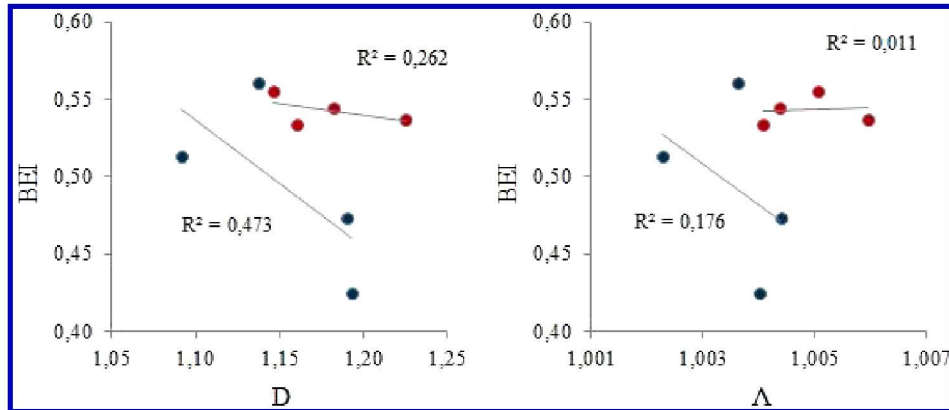
Relating the fractal dimension, that quantifies the hierarchical architecture of the drainage networks, to the BEI, that quantifies the heterogeneity of the soil PSD for the eight study sites, both parameters seem lineally independent ( $R^2 = 0.075$ ). By differentiating the study sites using bedrock materials some relationships emerge (Fig. 5). A negative relationship between  $D$  and BEI appears in both different lithologies. This weak relationship is not statistically significant but from a physical standpoint this negative trend was expected. Geomorphologists know that drainage networks developed

on coarse textures show lower drainage densities than drainage networks developed on finer textures. As coarse textures present higher values of BEI than fine textures, and fractal dimension and drainage density are closely related, it was expected that in areas with homogeneous bedrock lithology the values of the BEI, estimated for surficial soil samples, are lower when  $D$  increases. This behavior is more pronounced in the metamorphic rocks sites, while sites of plutonic rocks show more stability in values of BEI against changes in  $D$ .

Although the relationship is weak, the bedrock material emerges as a key environmental factor that constrains the architecture of the drainage networks and the particle soil distribution resulting in the fragmentation processes.

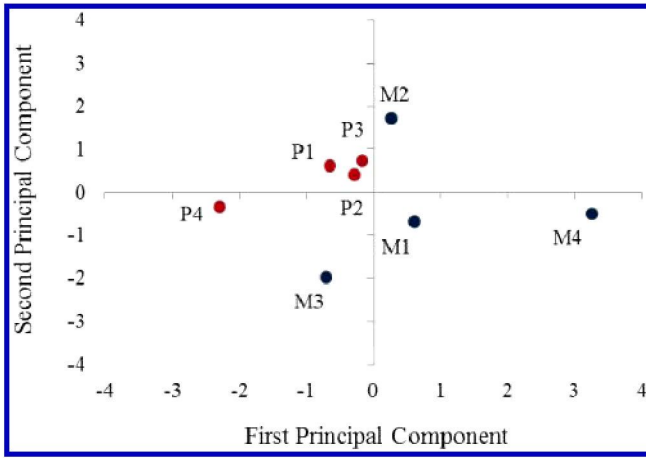
### 3.5. Principal Component Analysis

Multivariate analysis has been initially carried out with the 11 variables (Tables 2–4) described in Sec. 2. Variable “Order” has been removed from



**Fig. 5** Plots of the fractal dimension ( $D$ ) and the lacunarity ( $\Lambda$ ) of the drainage networks of the study sites versus the BEI of the particle-size distribution of the surficial soil samples of the study sites, showing the metamorphic lithologies (blue points) and the plutonic lithologies (red points).





**Fig. 6** Representation of the study sites on the first and second principal components.

this analysis due to its value of standardized kurtosis obtained in the monovariate analysis, which indicates that this variable has a distribution significantly deviated from normality. The correlation analysis shows that the following pairs of variables are significantly correlated: Altitude average and Slope average, Altitude average and  $\Lambda$ , Slope average and  $\Lambda$ , Drainage density and  $D$ ,  $D$  and  $\Lambda$ , BEI and Silt, BEI and Sand, and Silt and Sand. Aiming not to overvalue their weights in the principal components analysis, correlated variables have been removed. Finally, only five variables have been used in the analysis: Slope average, HI,  $D$ , BEI and Clay. First and second principal components contain the 74.6% of data variability. The weight of the variables in the principal components shows that first principal component is mainly defined by three variables (Slope average,  $D$  and Clay), while second principal component is mainly defined by the variable BEI. The proximity of the study sites with similar bedrock material in Fig. 6 suggests that explored soil and hydrological fractal parameters and properties, which describe the organization level of the system, could be the basis of statistical models to quantitatively classify bedrock materials or even to correct lithological maps.

#### 4. DISCUSSION

According to Clayton and Shamoon<sup>46</sup> who studied the resistance of rocks to erosion for the relative relief of Britain, the Pre-Cambrian metasediments are more resistant to erosion than the Pre-Cambrian granites, and our results seem to reaffirm that

finding in our study area. Higher values of average slope angle and HI, which in this work are shown in metamorphic units, are related in literature to harder bedrock materials.<sup>43,47</sup> Although granite seems to be more resistant to erosion than metamorphic rocks, except quartzite, the presence of biotite and the coarse grain in studied granites facilitate the erosion processes. Results of drainage density analysis agree with Strahler,<sup>15</sup> who reported similar values of drainage density for both plutonic and metamorphic lithologies.

The values of fractal dimension obtained in this work, ranging from 1.09 to 1.23, are lower than the values obtained by other authors.<sup>48,49</sup> However, according to Claps and Olivetto,<sup>50</sup> the more expected values of fractal dimension of river networks are those values close to 1.1, 1.5, and 1.7. These significant differences between our results and those reported in literature could be explained due to different methodologies to estimate the fractal dimension and different study scales. For us, the box sizes range employed in this work (320 to 20 m) is physically well chosen due to the nature of the drainage networks under investigation. According to Fig. 2, the use of box sizes bigger than 320 m would consider drainage networks as a continuous plane and fractal dimension would tend to the value 2. On the other hand, the use of box sizes smaller than 20 m would magnify the linear nature of the digitalized streams which compose the drainage network and the fractal dimension would tend to value 1.

The positive relationship between fractal dimension and lacunarity in drainage networks developed in homogeneous lithologies was recently affirmed.<sup>30</sup> Our present results reaffirm that finding.

The use of the BEI is probably the most innovative element of this work. One of the major advantages of the BEI is that it permits to discriminate between soil textures classified in the same textural class. The values of BEI obtained correspond to the same textural class in the USDA system, sandy loam, but by means of the BEI it has been possible to distinguish two groups inside this class, and each one of the studied bedrock lithologies is placed in a different group.

The aggregation of the individual data corresponding to the particle size distribution has being performed by the average value. Probably, further research considering some properties of the cartographic units of the soil map, as the area represented by each soil map unit within the study

site, could establish more and clearer connections between the parameters which quantify the complexity of the system at different spatial scales. As well, the inclusion in the analysis of the heterogeneity represented by the distribution of soil horizons along the soil profile could enrich the results.

Similar analysis on different bedrock lithologies in other regions would help to address the suitability of the proposed methodology at global scale.

## 5. CONCLUSION

The quantitative relationship between the hierarchical structure of the drainage networks at hillslope scale and the surficial soil texture at pedon scale has been investigated by means of fractal measures in 8 study sites (1.64 km<sup>2</sup>) distributed within areas of homogeneous bedrock lithology.

The automatic extraction of the drainage networks from medium resolution DEM has provided a starting point with quality enough to manifest environmental relationship at different scales, and suitable for the work objectives.

The fractal dimension ( $D$ ) and the lacunarity ( $\Lambda$ ) were the variables used to quantify the hierarchical structure of the drainage networks. The particle size heterogeneity was quantified using the BEI, introduced by Martin *et al.*<sup>36</sup> These three variables have demonstrated their capability to characterize and to quantify the complexity involved in natural system at different scales.

Although additional studies are still to be performed, the potential uses of BEI in Earth sciences are promising. This index is easily computable from textural data irrespective of the textural classification system and it provides a continuous textural parameterization that permits to numerically compare soil textures.

The fractal dimension of the drainage network shows a negative relationship with the BEI of the surficial soil samples. Thus, at least in both analyzed lithologies, finer textures are expected in drainage networks with higher values of fractal dimension.

Results suggest that the bedrock lithology which underlay the landscape constrains the architecture of the drainage networks developed on it and the particle soil distribution results of the fragmentation processes which affect it. Results of the multivariate analysis shows that the fractal dimension of

the drainage network and the BEI of the particle-size distribution could be useful parameters to quantitatively classify bedrock materials.

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